

Book Review

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“Algorithms for Worst-Case Design and Applications to Risk Management” by Berc Rustem and Melendres Howe (Princeton, NJ: Princeton University Press, 2002) ISBN 0-691-09154-4.

In the fourth quarter of 1997, Long-Term Capital Management, Inc. (LTCM) returned \$2.7 billion to investors, an annualized rate of return of about 40 percent. By the middle of 1998, LTCM had more than \$125 billion under management, phenomenal growth for a firm that was founded in 1993. The key to such rich returns and rapid growth was leverage, and the key to leverage was the low operating risk promised by the arbitrage plays that LTCM was reportedly pursuing. As everyone knows, the next few months things did not work out the way LTCM’s principals, including two Nobel laureates, had planned. One way to interpret what went wrong at LTCM was that the firm’s managers believed in their models too much. They probably would have been well served by a good hard look at the tools of Berc Rustem and Melendres Howe.

Rustem, Professor of Computing at Imperial College, London, and co-author Howe of the Asian Development Bank set out to explain how financial engineers can guarantee good returns in an uncertain world while taking off the table the worst-case outcomes like those experienced by LTCM’s shareholders. In words, the decision maker sacrifices some returns in the world represented by her model in exchange for eliminating worst-case worlds from contention. Unlike a mean-variance optimizer, who is willing to assume a parametric characterization of the trade-off between expected rates of return and the variance around that expectation, the world that Rustem and Howe are interested in entertains the idea that the model under study is misspecified in a way that defies parametric measure. The decision maker is assumed to be able specify a set of possible true models, but no probabilities are assigned to one model versus another. Without a parametric notion of what this misspecification might look like, Gilboa and Schmeidler (1989) have shown that a minmax problem is the natural modeling environment. This differs in interesting ways from standard, Value-at-Risk modeling.

As the authors note in Chapter 1, the way this is formalized is:

$$\min_{x \in \mathcal{X}} \max_{y \in \mathcal{Y}} f(x, y)$$

where $x \in \mathcal{X}$ is the decision maker's set, and $y \in \mathcal{Y}$ is a vector of uncertain variables defined over a feasible set, \mathcal{Y} . Metaphorically, one way to think about this formulation is to imagine a decision maker playing a game against a hostile opponent, "nature," with our decision maker attempting to minimize losses while nature attempts to maximize them.

Problems of this nature do not necessarily render nice, differentiable functions or unique equilibria. Without gradient vectors defined everywhere, specialized algorithms are required and hence this book.

The book is replete with mathematical proofs, but it is not primarily a book about theory. This is very much a tool box book; algorithms for solving a particular set of problems. The problems that interest the authors are invariably static, if possibly repeated. Connections to the literature on robust control and entropy theory are eschewed. Those interested in robust approaches to dynamic maximization problems will have to look elsewhere. However, if the book is not particularly broad, it is plenty deep and thorough. What it does, it does very well.

Chapter 1 introduces the basic concept of minmax. Chapters 2 through 7 outline the algorithms for solving such problems, first in general terms, and then more specifically. The set \mathcal{Y} may be either discrete or continuous and there are different algorithms for each. Chapters 2 to 5 focus on the continuous case leaving chapters 6 and 7 for the discrete case. In many instances, discrete-case problem can be rewritten as a continuous one, but since the computational cost of solving discrete-case problems is usually lower than those of continuous problems, specialized algorithms are advanced for both. Chapters 8 through 11 provide detailed applications to problems in financial management. Every chapter has the admirable feature of being written on many levels, allowing readers of every level of sophistication to get something out of what she is reading. Time is taken to briefly explain elementary financial theory and terminology without sacrificing rigor later on.

Rustem and Howe do not expend a lot of effort to justify the minmax criteria, a reflection, perhaps, of the engineering-cum-computation orientation of the book. Since the minmax criteria is not without its critics, some defense might have been useful. In effect, though, the defense arrives implicitly in the form of the cleverness of the applications. Critics of minmax argue that

as uncertain (in the sense of Knight) one might be, the division of the world into possible, $y \in Y$, and impossible, $y \notin Y$, worlds, but without any idea of likelihood in between, is too stark. In their applications chapters, Rustem and Howe combine Bayesian (parametric) risk management with minmax (non-parametric) in practical ways that take advantage of both thus allowing the simultaneous consideration of quantifiable and unquantifiable risks. To my taste, this is the book's greatest strength.

An example of this is Chapter 8, where the authors lay out in detail continuous minmax strategy for hedging for call options. As most readers of this review will know, a call option is the right, but not the obligation, to buy an asset at a fixed price on or before a pre-determined date. The writer (seller) of a call option, if she doesn't cover her position by, say, also holding the underlying asset, accepts an open position with which she could theoretically incur infinite losses. One traditional method for managing a short position in a call option, the so-called delta hedging method, is to adjust the extent to which the option position is covered according to the "delta" of the option; that is, the derivative of the value of the position with respect to a change in the price of the underlying asset. This method builds on the famous Black-Scholes option pricing formula and its offspring. That formula, however, depends on a credible estimate of the (assumed constant) instantaneous standard deviation of the returns to the underlying asset. The authors are complete in their description of the problem, beginning with very basic ideas, laying out the delta method and its deficiencies, and then overlaying the minmax hedging strategy as a complement to delta hedging. They take explicit consideration of transactions costs, something from which more theoretical presentations typically abstract. Several worst-case scenarios are considered, a simulation exercise is presented as well as some "limited" empirical results. Because the various minmax proposals are compared against the pure delta hedging results, the reader (or the would-be financial engineer more generally) can judge for herself the efficacy of minmax as an incremental strategy.

Chapter 9 examines minmax applied to asset allocation problems. Standard applications of asset allocation problems involve the estimation of expected returns and the covariance of those returns across a wide variety of assets, together with the application of mean-variance optimization procedures to pick portfolios that efficiently trade-off expected return against the variance of return. As valuable as these methods are, they depend critically on accurate estimates

of expected returns and the covariance matrix of those returns. The cleverness here is two-fold: first, the authors' application of their tools to returns relative to a benchmark portfolio; and second, they consider robustness to rival models of either expected returns, risks, or both. This set up allows the standard mean-variance optimization techniques to be applied on each scenario and then minmax is applied over top to protect against the worst-case scenario. Once again, applications are presented, this time to bond portfolio selection, and extensions are discussed.

Chapter 10 turns to asset and liability management under uncertainty. This chapter considers how to manage such problems as how a pension fund would hedge expected future payouts through the selection of assets the risky returns to which would counter the risks of the payouts. This is usually done with either cash matching—picking the terms to assets match those of liabilities—or portfolio immunization—picking assets and liabilities together such that the derivative of the present value of the entire portfolio with respect to a change in some “exogenous” variable is minimized. Cash matching is a passive strategy in that it does not require shifting of portfolio assets unless liabilities shift, while immunization requires active rebalancing of the portfolio to changes in conditions. Both methods, but particularly portfolio immunization, depend on parametric measures of asset value responses to changes in exogenous variables. The minmax strategies discussed by the authors protect against model misspecification of a fairly general type; in fact, if there is a problem with what the authors propose it is the paucity of guidance in deciding against what the decision maker should protect. This, however, is a general issue with worst-case design and the authors do as well as anyone in suggesting practical methods for getting a handle on these problems. In this chapter, as in the previous one, several applications and extensions are considered.

The final chapter is on robust currency management, a problem of some magnitude for multinational corporations whose costs and revenues are generated in many currencies but whose returns are measured in just one. Reflecting the practical nature of the book, the authors devote non-trivial space to reviewing the case where any hedging at all of currency risk is warranted before moving on to discussing strategic and tactical approaches to risk management. The expected exchange rate is assumed to be a function of a vector of variables, plus an error term. The error associated with each argument is taken as a part of Y . Noting that point forecasts of exchange rates are notoriously unreliable, the authors sensibly advocate range forecasting and

then protecting against the lower bound of that range as a worst case.

Rustem and Howe is not without its limitations. As noted previously, all the methods are static, although in some cases multi-period static problems are discussed. And I would have liked to see more discussion of how one chooses the worst case against which one would protect; the argument to which the authors allude, but do not make explicit, that the historical record provides guidance on what is in Y seems a weak one to me since the rarity of worst-case scenarios is the reason that parametric risk assessment is deemed inadequate. Perhaps there really is no alternative to “wisdom” as the authors note in the Preface. I also would have liked to have seen a comparison of the recent crop of linear matrix inequality methods to the minmax solutions the authors advance. Finally, the provision of some of the code the authors use would have been a nice touch. But I hasten to say that these are mere quibbles.

In the end, the Rustem and Howe is a very solid, cohesive treatment of worst-case avoidance applied to financial risk management, combining theoretically sound methods in satisfyingly practical ways. Anyone with an interest in real-world financial risk management would be making an error—perhaps not of LTCM-sized magnitude, but large nonetheless—by not having it on her bookshelf.